Formulae of Tree Height Curve and Volume Curve Derived from Theory of Column Buckling

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Abstract In this paper, the new formulae of tree height curve and volume cdrve were derived from the theory of column buckling. They were applied to artificial Pine (*Pinus sylvestris* var. *mongolica*) and Larch (*Larix principis rupprechtii*). The results demonstrated that the new formulae were more effective and precise than conventional formulae of height curve and volume curve.

Key words: Column buckling theory, Tree height curve, Volume curve

Introduction

In general, the cuver of relationship between the tree height and the diameter at breast height (dbh) is considered as height-dbh curve or as tree height curve (h-d). There are two methods to set up the formulae of tree height curve: one is graphic method; the other is mathematical equation by selecting applicable regression equation. The two methods are based on data from investigation in forest. Moreover, they can not provide the theoretical explanation of the relationship between the tree height and its dbh. For this reason, this paper tries to explain the inherent relationship between the tree height and dbh, then build the volume formula based on height curve. Finally the precision and feasibility of the tree height curve and volume curve used for artificial Pine (*Pinus Sylvestris* var. *mongolica*) and Larch (*Larix Principis-rupprechtii*) forests were examined.

Methods and Data

According to the theory of column buckling [1], the relationship between the diameter (d) and the height (h) of a column must meet:

$$h^3/d^2 < 2E/(pg) \quad (1)$$

where, E is elasticity models of the wood;
P is the density of the wood;
g is acceleration of gravity.

If there is no enough basal cut area (or diameter) to support the tree stem for standing erectly, it would be bent by its above weight or would be deformed of fractured because of outside factors (wind, snow, etc.). Therefore, there must exist intrinsic relationship between diameter and height of a tree for supporting its weight and to maintain erect.

The wood models (E) and the density (p) vary with tree species, but at the same region for the same tree species, E and p, and g are constant. Thus the right side of the above equation (1) is constant, i.e.

where if $a' = 2E/(pg)$, then

$$h^3/d^2 < a' \quad (2)$$

$$\text{or } h^3 = a'd^2 \quad (3)$$

According to the above definition, we can obtain the relationship of the tree height *(H)* and its dbh (D), that is h-d curve to be as follows

$$H = ad^{2/3} \quad (4)$$

Where, a is the estimated prameter, $a < a'^{1/3}$. The general form of stem volume (V) curve is as follows:

$$V = (\pi/4) f_{1.3} D^2 \mathrm{H} \quad (5)$$

where $f_{1.3}$ is form factor at breast height. Therefore, substituting equation (4) into equation (5), we can get stem volume curve as follows

$$V = (\pi/4) f_{1.3} D^2 aD^{2/3} = (\pi/4) f_{1.3} aD^{8/3} \quad (6)$$

Where if $b = (\pi/4) f_{1.3}$, then

$$V = bD^{8/3} \quad (7)$$

794 Larch and 754 Pine trees were selected randomly from Saihanba Mechanical Farm in Hebei Province respectively. They were measured at the dbh, and the

different diameter at i/10 (i=0,1,... 9) of the tree height from ground to tree tip, and the total tree height after felling. The individual tree volume was calculated by sectional measurement :

$$V=0.2H(G_{0.1}+G_{0.3}+G_{0.5}+G_{0.7}+G_{0.9}) \quad (8)$$

Where, $G_{0.1}$ is basal cut area at $i/10$ of tree height.

The compiling data were tested using 3 times standard deviation of mean normal form number. That is because the variation of normal form factor is very small and the stem form for the same species at the same region is very similar to each other. By testing, no abnormal data were found.

Results

We simulate equation (4) and equation (7) in ordinary Least-square method. Meanwhile, in order to compared conventional method we also simulate formulae of the tree height curve and volume curve in conventional method. The height curve and volume curve derived form the theory of column buckling are as follows

Larch $H=1.9877D^{2/3}$ (r-0.845) (9)

$V=0.0144+6.6337E\text{-}5D^{8/3}$ (r=0.976) (10)

Pine $H=1.5206D^{2/3}$ (r=0.905) (11)

$V=0.0156+4.6710E\text{-}5D^{8/3}$ (r=0.996) (12)

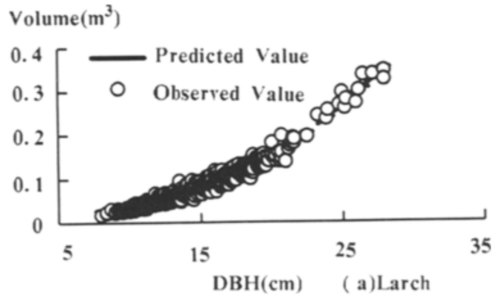

Fig. 1. Height curve derived from theory of column buckling

The results of simulation are plotted as Fig. 1, 2, 3, and 4. We selected seven conventional empirical regression equations to model the monadic volume formula [2], of which, we choose one with the largest correlation index and the least sum of relative error as the optimal model of conventional volume formula.

Larch $V=6.0068E\text{-}5D^{1.8263}H^{1.0423}$ (r=0.857) (13)

Pine $V=9.6605E\text{-}5D^{1.8146}H^{0.8276}$ (r=0.996) (14)

Using the same method, we choose the one curve with largest correlation index from the four of common tree height-diameter empirical regression equations (2) as the optimal model,

Larch $H=1.6122D^{0.7483}$ (r=0.857) (15)

Pine $H=2.4474D^{0.5029}$ (r=0.905) (16)

Therefore, substituting equation (15) and (16) into equation (13) and (14), respectively, we can obtain the one-way volume formula as follows:

Larch $V=6.0068E\text{-}5D^{1.8263}(1.6122D^{0.7483})^{1.0423}$ (17)

Pine $V=9.6605E\text{-}5D^{1.8146}(2.4474D^{0.5029})^{0.8276}$ (18)

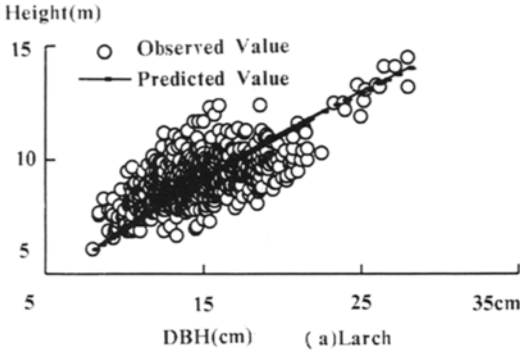

Fig. 2. Volume curve derived from theory of column

Table 1. Comparison of error with two height curves

Equation	Man	Variance (m)	Range (m)	t-value (m)
		Larch		
9	0.0251	0.055	3.6	0.845
15	0.834	0.0575	3.9	0.857
		Pine		
11	0.0589	0.9085	4.9	0.065
16	-0.2598	0.7333	5.1	-0.354

We calculated the difference of actual tree height (H) and estimated height (H') from height curves as the individual tree height error (Table 1). The t-value of equation (9) and (15) for Larch, are 0.845 and 0.847. t-value of equation (11) and (16) for Pine are 0.065 and -0.354, respectively. All of them are less than $t_{0.05}$=1.96. Difference is not significant, so the equations are feasible. The mean individual tree height error of equation (15) is 3 times as much as that of equation (9) for Larch, and equation (16) is 5 times as much as that of equation (11) for Pine. The precision of equation (11)

(99.3%) for Pine is slightly better than that of equation (16) (99.0%). i.e. the height curve based on column bucking theory is better than conventional height curve. Actual volume (v) is calculated in sample tree sectional measurement and volume (v′) is calculated in volume curves. Three Formulae of volume curves, that is the new volume curve, conventional binary and monadic volume formula.

Table 2. Comparison of error with the volume curves

Equation	Mean (m)	Variance (m)	Range (m)	t-value
		Larch		
10	-5.66E-05	6.79E-04	0.052	-0.083
13	3.33E-04	2.07E-04	0.067	1.234
17	-2.44E-03	9.42E-04	0.069	-2.590
	Pine			
12	6.03E-07	1.23E-04	0.065	0.005
14	-1.12E-04	7.80E-03	0.073	-0.014
18	-6.52E-03	2.30E-03	0.073	-2.835

We calculated the individual sample tree volume error (v-v′), the results were listed in Table 2. According to the results of t-test, the t-value of equation (10) and (13) for Larch are -0.083 and 1.234, and for Pine are 0.005 and -0.014, respectively. All of them are less than $t_{0.05}$=1.96.

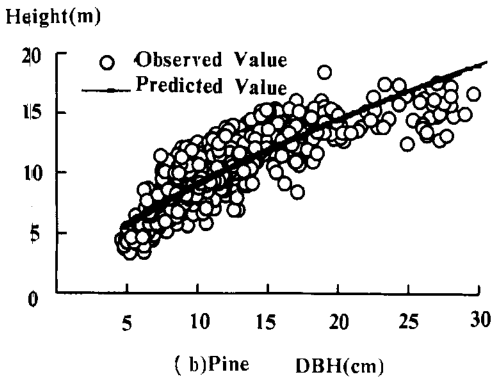

Fig. 3. Height curve derived from theory of column buckling

The difference is not significant, both of them are applicable. But the binary volume formula is not applicable, because the t-value of equation (17) and (18) are -2.59 and 2.83 respectively. All of them are larger than 1.96. The mean and variance of volume error of equation (12) and (14) are 96.5% and 95.6% respectively for Pine. From this point, we can conclude that the volume curve based on column buckling theory is better than conventional binary and monadic volume formula.

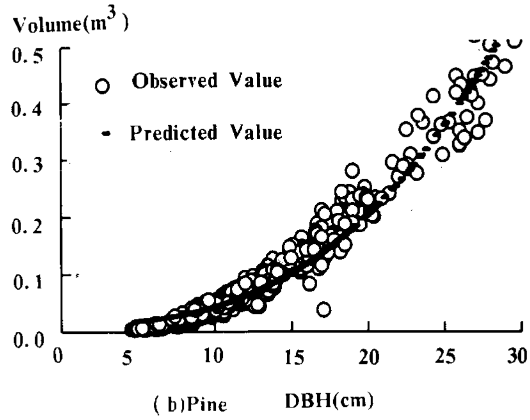

Fig. 4. Volume curve derived from theory of column

Conclusion

The above results indicate that the formulae of height curve and volume curve based on column buckling theory are ideal equations, and the previsions are better than conventional formulae of height curve and volume curve. The index of dbh are fixed as 2/3 and 8/3. The height curve and volume curve based on column buckling theory, comparing with conventional height curve and volume curve, is very simple, few parameters and easy modeled. Meanwhile, it is convenient to compare the height curve and volume curve of different tree species and different stands in terms of parameter *a* and *b*. This study shows that the parameter *a* of Larch (1.9877) is larger than that of Pine (1.5206) at Saihanba Mechanical Farm, i.e. if dbh is identical, the tree height of the former is 1.3 times as much as that of the latter.

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